

# Efficient faithful qubit transmission with frequency degree of freedom\*

Xi-Han Li<sup>a</sup>, Bao-Kui Zhao<sup>a</sup>, Yu-Bo Sheng<sup>a</sup>, Fu-Guo Deng<sup>a,b†</sup>, Hong-Yu Zhou<sup>a</sup>

<sup>a</sup>*Key Laboratory of Beam Technology and Material Modification of Ministry of Education, and College of Nuclear Science and Technology, Beijing Normal University, Beijing 100875, China*

<sup>b</sup>*Department of Physics, Beijing Normal University, Beijing 100875, China*

(Dated: August 17, 2009)

We propose an efficient faithful polarization-state transmission scheme by utilizing frequency degree of freedom besides polarization and an additional qubit prepared in a fixed polarization. An arbitrary single-photon polarization state is protected against the collective noise probabilistically. With the help of frequency beam splitter and frequency shifter, the success probability of our faithful qubit transmission scheme with frequency degree of freedom can be  $1/2$  in principle.

PACS numbers: 03.67.Pp, 03.67.Hk

## I. INTRODUCTION

The main task of quantum communication is transmitting information between two distant parties. However, during the transmission, the quantum information carriers are usually infected by various things, which we call noise. By far, various methods have been proposed to solve the problem caused by noise. For example, phase coding was introduced [1] to overcome the influence on the polarization of photons by thermal fluctuation, vibration and the imperfection of the fiber. Corresponding to different types of noise, some other methods are proposed such as quantum error correct code (QEC) [2], error rejection [3, 4, 5] and decoherence-free subspace (DFS) [6, 7, 8, 9]. Although QEC and DFS methods can be used to suppress the effect of noise effectively, they are sensitive to channel losses and need much resource.

As the noise usually fluctuates slowly in time, there is an important precondition for dealing with it in quantum information processing, which called a collective noise assumption [10]. That is, if several photons travel through the collective-noise channel simultaneously or the maximum interval between them is shorter than the variation of noise, the alteration caused by noise is the same one for each qubit. Recently, some error-rejection qubit transmission schemes have been proposed by using only one or two photons with linear optics. In 2005, Kalamidas proposed two single-photon transmission schemes to reject arbitrary errors [3]. The average success probability of the second protocol is 100%. However, fast polarization modulator (Pockels cell), whose synchronization makes it difficult to be implemented with current technology, are employed. Subsequently, we presented a single-photon transmission scheme against collective noise with only passive linear optical elements [4]. In 2005, Yamamoto et al proposed a qubit distribution scheme against collective noise with the help of one additional qubit in a

fixed polarization [5]. We call it PRL95 protocol below, whose experimental results with a collective dephasing noise are published in Ref.[11]. The total success probability can be improved from  $1/16$  to  $1/8$  with "deterministic two-qubit operations", which was not presented clearly in their work [5]. At present, a deterministic two-qubit operation based on linear optics is difficult to be implemented in practice as it can be done with only a very low efficiency [12].

In recent years, some other degrees of freedom (DOFs) of photons besides their polarization attract much attention, such as the frequency [13], spatial mode [14, 15], orbital angular momentum (OAM)[16], transverse spatial mode [17] of photons, and so on. Additional to polarization, other DOFs can be used to implement complete Bell-state analysis [18], entanglement purification [13, 14, 15], superdense coding [19], and so on. As the frequency DOF is stable in any transmission surroundings, it was used to code message in quantum key distribution schemes and some good results were obtained [20, 21, 22, 23]. Photon with more than one degree of freedom (DOF) can be prepared with current technology easily [24].

Quantum state transmission is important to quantum information process as the transmission of qubits carrying information is the first step of quantum communication. In this paper, we proposed an efficient faithful qubit transmission scheme assisted by an additional qubit which has different frequency with the signal one. The frequency DOF is used to mark these two photons. The success probability of our protocol is eight times of PRL95 protocol at best without using the deterministic two-qubit operation. We also find that the success probability of PRL95 scheme can be improved from  $1/16$  to  $1/8$  just by adding a half-wave plate (HWP).

## II. EFFICIENT FAITHFUL QUANTUM STATE TRANSMISSION WITH FREQUENCY DEGREE OF FREEDOM

The principle of our scheme for efficient faithful quantum state transmission with frequency degree of freedom

\*Published in Opt. Commun. 282 (2009) 4025-4027

†Author to whom correspondence should be addressed.

Email address: fgdeng@bnu.edu.cn

is shown in Fig.1. An arbitrary single-photon pure state to be transmitted can be written as

$$|\psi\rangle_s = \alpha|H\rangle_s + \beta|V\rangle_s, \quad (|\alpha|^2 + |\beta|^2 = 1). \quad (1)$$

Here  $|H\rangle$  and  $|V\rangle$  represent the horizontal and the vertical polarization states, respectively. The subscript  $s$  represents the signal photon. Alice prepares a reference photon in the state  $(1/\sqrt{2})(|H\rangle_r + |V\rangle_r)$ . The frequencies of the reference photon and the signal one are  $\omega_r$  and  $\omega_s$ , respectively. Two photons with different frequencies can be prepared with current technology [20, 23, 24]. The input state of the quantum system composed of the two photons can be written as

$$|\psi\rangle_{rs} = \frac{1}{\sqrt{2}}(|H_r, \omega_r\rangle + |V_r, \omega_r\rangle) \otimes (\alpha|H_s, \omega_s\rangle + \beta|V_s, \omega_s\rangle).$$

For the sake of simplicity,  $\omega_r$  and  $\omega_s$  can be omitted in the following, i.e.,

$$|\psi\rangle_{rs} = \frac{1}{\sqrt{2}}(|H_r\rangle + |V_r\rangle) \otimes (\alpha|H_s\rangle + \beta|V_s\rangle). \quad (2)$$

The subscript  $r$  and  $s$  can mark the frequency difference between the two photons. Each photon is split into two pulses by the first polarizing beam splitter (PBS<sub>1</sub>), which drives  $|H\rangle$  going through channel 1 and  $|V\rangle$  through channel 2. Notice that the two photons are sent to Bob simultaneously in this protocol. These two noise channels have equal length and the phase shift of these two channels is consistent. The alternation caused by noise is

$$\begin{aligned} |H\rangle_{r,s} &\rightarrow \delta_1|H\rangle_{r,s} + \eta_1|V\rangle_{r,s}, \\ |V\rangle_{r,s} &\rightarrow \delta_2|H\rangle_{r,s} + \eta_2|V\rangle_{r,s}. \end{aligned} \quad (3)$$

Parts of each photon converge at PBS<sub>2</sub>.

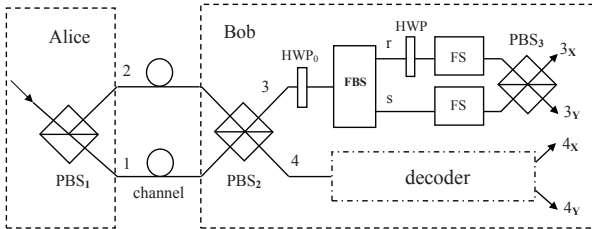


FIG. 1: Schematics of qubit distribution protocol with frequency degree of freedom. Two photons with different frequencies  $\omega_r$  and  $\omega_s$  are sent to Bob simultaneously. The noise channels 1 and 2 have the same length. Frequency beam splitter (FBS) is used to divide photons with different wavelengths into different spatial modes, i.e. the reference photon with the frequency  $\omega_r$  goes to the up path and the signal one with the frequency  $\omega_s$  goes to the down path. Frequency shifter (FS) is used to eliminate the frequency distinguishability. The decoder set in port 4 is same as the one in port 3, which is indicated by the dash-dotted rectangle.

The state of the quantum system composed of the two photons after the PBS<sub>2</sub> becomes

$$\begin{aligned} |\psi_1\rangle = & \frac{1}{\sqrt{2}}[\alpha\delta_1^2|H\rangle_{r3}|H\rangle_{s3} + \beta\eta_2^2|V\rangle_{r3}|V\rangle_{s3} \\ & + \delta_1\eta_2(\alpha|V\rangle_{r3}|H\rangle_{s3} + \beta|H\rangle_{r3}|V\rangle_{s3}) \\ & + \alpha\eta_1^2|V\rangle_{r4}|V\rangle_{s4} + \beta\delta_2^2|H\rangle_{r4}|H\rangle_{s4} \\ & + \eta_1\delta_2(\alpha|H\rangle_{r4}|V\rangle_{s4} + \beta|V\rangle_{r4}|H\rangle_{s4}) \\ & + \alpha\delta_1\eta_1(|H\rangle_{r3}|V\rangle_{s4} + |V\rangle_{r4}|H\rangle_{s3}) \\ & + \delta_1\delta_2(\alpha|H\rangle_{r4}|H\rangle_{s3} + \beta|H\rangle_{r3}|H\rangle_{s4}) \\ & + \beta\delta_2\eta_2(|H\rangle_{r4}|V\rangle_{s3} + |V\rangle_{r3}|H\rangle_{s4}) \\ & + \eta_1\eta_2(\alpha|V\rangle_{r3}|V\rangle_{s4} + \beta|V\rangle_{r4}|V\rangle_{s3})]. \end{aligned} \quad (4)$$

Here 3 and 4 are two output ports of PBS<sub>2</sub>. A HWP whose orientation is 45° is placed in the path 3 behind the PBS<sub>2</sub>, which acts as the  $\sigma_x$  operation

$$\begin{aligned} |H\rangle_{r,s} &\rightarrow |V\rangle_{r,s}, \\ |V\rangle_{r,s} &\rightarrow |H\rangle_{r,s} \end{aligned} \quad (5)$$

The total state before the decoder becomes

$$\begin{aligned} |\psi_2\rangle = & \frac{1}{\sqrt{2}}[\alpha\delta_1^2|V\rangle_{r3}|V\rangle_{s3} + \beta\eta_2^2|H\rangle_{r3}|H\rangle_{s3} \\ & + \alpha\eta_1^2|V\rangle_{r4}|V\rangle_{s4} + \beta\delta_2^2|H\rangle_{r4}|H\rangle_{s4} \\ & + \alpha\delta_1\eta_1(|V\rangle_{r3}|V\rangle_{s4} + |V\rangle_{r4}|V\rangle_{s3}) \\ & + \beta\delta_2\eta_2(|H\rangle_{r3}|H\rangle_{s4} + |H\rangle_{r4}|H\rangle_{s3}) \\ & + \delta_1\eta_2(\alpha|H\rangle_{r3}|V\rangle_{s3} + \beta|V\rangle_{r3}|H\rangle_{s3}) \\ & + \eta_1\delta_2(\alpha|H\rangle_{r4}|V\rangle_{s4} + \beta|V\rangle_{r4}|H\rangle_{s4}) \\ & + \delta_1\delta_2(\alpha|H\rangle_{r4}|V\rangle_{s3} + \beta|V\rangle_{r3}|H\rangle_{s4}) \\ & + \eta_1\eta_2(\alpha|H\rangle_{r3}|V\rangle_{s4} + \beta|V\rangle_{r4}|H\rangle_{s3})]. \end{aligned} \quad (6)$$

One can see the last eight terms are preserved against the noise and all of them have the same form  $\alpha|H\rangle_{ri}|V\rangle_{sj} + \beta|V\rangle_{rj}|H\rangle_{si}$  ( $i, j \in \{3, 4\}$ ) excepts the spatial modes. Bob's decoder, which is used to eliminate the difference between these two photons and extract the initial quantum state, is made up of frequency beam splitter (FBS), frequency shifter (FS) and PBS. The FBS makes the photons with the frequency  $\omega_r$  and  $\omega_s$  go through two different paths directly [25]. This operation can also be implemented by wavelength-division-demultiplexing (WDM) [13] or fiber bragg grating (FBG) [22, 23].

After the operation of HWP placed on the up path, the frequencies of these two photons will be manipulated to a same value set in advance by means of the frequency shifter (FS). The modulation of the frequency can be carried out by acousto-optic modulator (AOM) [25, 26], sum-frequency generation (SFG) process [27], and so on. Then the distinguishability of these two photons are eliminated and the interference could take place at the PBS<sub>3</sub>. The ultimate form of the total state is

$$\begin{aligned} |\psi_3\rangle = & \frac{1}{\sqrt{2}}[\alpha\delta_1^2|H\rangle_{3y}|V\rangle_{3y} + \beta\eta_2^2|V\rangle_{3x}|H\rangle_{3x} \\ & + \alpha\eta_1^2|H\rangle_{4y}|V\rangle_{4y} + \beta\delta_2^2|V\rangle_{4x}|H\rangle_{4x} \end{aligned}$$

$$\begin{aligned}
& +\alpha\delta_1\eta_1(|H\rangle_{3y}|V\rangle_{4y} + |H\rangle_{4y}|V\rangle_{3y}) \\
& +\beta\delta_2\eta_2(|V\rangle_{3x}|H\rangle_{4x} + |V\rangle_{4x}|H\rangle_{3x}) \\
& +\delta_1\eta_2(\alpha|V\rangle_{3x}|V\rangle_{3y} + \beta|H\rangle_{3y}|H\rangle_{3x}) \\
& +\eta_1\delta_2(\alpha|V\rangle_{4x}|V\rangle_{4y} + \beta|H\rangle_{4y}|H\rangle_{4x}) \\
& +\delta_1\delta_2(\alpha|V\rangle_{4x}|V\rangle_{3y} + \beta|H\rangle_{3y}|H\rangle_{4x}) \\
& +\eta_1\eta_2(\alpha|V\rangle_{3x}|V\rangle_{4y} + \beta|H\rangle_{4y}|H\rangle_{3x})]. \quad (7)
\end{aligned}$$

Here  $x$  and  $y$  are the two spatial modes of each PBS on the paths 3 and 4. We omit the subscript  $r$  and  $s$  as the frequency distinguishability of these two photons is erased and they are indistinguishable when they arrive at PBS<sub>3</sub> and PBS<sub>4</sub>. From Eq.(7), we can select the instances that two photons arriving at different spacial mode  $x$  and  $y$  ( $3x/3y$ ,  $4x/4y$ ,  $3x/4y$  and  $4x/3y$ ) and measure one photon with  $X$  basis, and then the other photon can be manipulated to the initial state with a proper unitary operation chosen according to the measurement result. For example, if the outcome of the measurement on the photon coming from  $3y$  is  $|+x\rangle_{3y} = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ , the photon in the output  $3x$  is in the state  $\alpha|V\rangle + \beta|H\rangle$  and Bob can obtain the original state  $\alpha|H\rangle + \beta|V\rangle$  with a bit-flipping operation  $\sigma_x = |H\rangle\langle V| + |V\rangle\langle H|$ ; If Bob obtains the outcome  $|-x\rangle_{3y} = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$ , the photon in the output  $3x$  is in the state  $\alpha|V\rangle - \beta|H\rangle$  and Bob can obtain the original state  $\alpha|H\rangle + \beta|V\rangle$  with the operation  $-i\sigma_y = |H\rangle\langle V| - |V\rangle\langle H|$ .

The success probability of this transmission scheme is  $\eta^2/2$  with the help of the frequency DOF, where  $\eta$  is the efficiency of a FS. We also can simplify this protocol by using only one FS for one decoder. That is keeping the frequency of the signal qubit and adjusting the frequency of reference one to  $\omega_s$ . In this way, the success probability is  $\eta/2$ , which can be  $1/2$  at best with the efficient operation of FS.

### III. DISCUSSION AND SUMMARY

The main part of this qubit transmission scheme is the decode process, in which two photons with different frequencies should be guided to two different pathes faithfully and be reset to have the same frequency by FS. The FS can be implemented by several means with current technique, such as acousto-optic modulator (AOM) [25, 26], sum-frequency generation (SFG) process [27], and so on. And sufficient high modulation efficiency is accessible. It is reported in Ref. [28], the internal conversion efficiency of a SFG process is 99% and the overall efficiency can be 65%. Moreover, the frequency difference between the signal photon and the reference one should be set to a small value which could be discriminate by the FBS effectively and the noise effects on these two photons with different frequencies can be viewed as identical.

Compared with PRL95 protocol, we use the frequency DOF to mark these two photons instead of temporal DOF. The success probability of our protocol can be eight times of PRL95 scheme at best, whose success probab-

ility is only  $1/16$  without resorting to deterministic two-qubit operation. The improvement comes from two parts. First, a HWP is introduced in path 3 before the decoder, which is utilized to extract the original state from terms that each output of PBS<sub>2</sub> has one photon [5]. With the use of HWP<sub>0</sub> shown in Fig.1, the success probability of PRL95 scheme can be improved from  $1/16$  to  $1/8$  without any deterministic two-qubit operation. Second, the use of frequency DOF avoids the 75% loss during the eliminating of the distinguishability of these two photons.

Compared with the quantum state transmission scheme with single photons [4], our protocols have no predominance in terms of success probability. However, schemes with other DOF requires a much smaller order of timing precision. Moreover, as the two photons in cases we selected ( $x/y$ ) are always in the same pure entangled state  $\alpha|VV\rangle + \beta|HH\rangle$ , the scheme using additional qubit can also be used to prepare entangled states remotely in Bob's lab without classical communication with the collective noise. This protocol can be used not only to transmit a pure single-particle state, but also to transmit a mixture state.

In the quantum state transmission scheme with an additional qubit, an assistant DOF, which is stable during the transmission, is needed to differentiate the two photons for the sake of performing different operations on each photon after the noise. Subsequently, the DOF should be erased efficiently for extracting the uncorrupted polarization state we want. The frequency DOF satisfies these restrictions in principle. With the development of experiment technique, more suitable DOF may be found in future and the implementation of this theoretic protocol would be more efficient.

In summary, we have proposed an efficient faithful qubit transmission scheme against collective noise with frequency DOF. The success probability can be  $1/2$  in principle, which is eight times of success probability of PRL95 scheme utilizing the temporal DOF. We also find a HWP is enough to improve success probability of PRL95 from  $1/16$  to  $1/8$  without using deterministic two-qubit operation. The protocol we present is a theoretic frame and the success probability of  $1/2$  can be obtained with the development of experiment techniques. Maybe some other DOFs which are antinoise and easy to control will be found in the future. A suitable DOF for assisting the transmission of polarization state can be chosen according to the practical condition.

### ACKNOWLEDGEMENTS

This work is supported by the National Natural Science Foundation of China under Grant No. 10604008, a Foundation for the Author of National Excellent Doctoral Dissertation of China under Grant No. 200723, and Beijing Natural Science Foundation under Grant No. 1082008.

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